

# **A Geometric Model Relating Schwarzschild and FRW Spacetimes**

Ingo Franz <if@ehotel.de> <ingofranz@gmx.net>

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Prerequisites are a solid knowledge of the Lorentz transformation, as well as a basic understanding of the line element and the Schwarzschild metric.

## Abstract

A geometric model is presented that allows for a continuous mapping between a black hole described by the extrapolated interior Schwarzschild metric and an interior cosmological spacetime described by the Friedmann–Robertson–Walker metric. The focus is placed on the initialization of this correspondence, which is characterized geometrically by a reversal of the curvature direction at the event horizon.

The metric employed for the transition is formally derived, and it is shown how a consistent transformation between the exterior Schwarzschild geometry and an interior FRW spacetime can be formulated. The model is developed entirely within the framework of general relativity and does not challenge the cosmological standard model, but rather provides a complementary geometric interpretation within this established framework.

In this sense, the construction may be regarded as providing a geometric basis for a possible junction description between static and dynamic spacetime during the gravitational collapse.

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# 1 Historical Context of Motion-Based Models

The idea of interpreting cosmological redshift as the result of real motion through space is not new. Edwin Hubble initially interpreted the recession of galaxies in the classical sense as linear motion. Later, Edward Arthur Milne (1935) developed a kinematic cosmology based on special relativity, in which galaxies recede from a common origin in a pre-existing spatial framework.

These approaches differ conceptually from relativistic cosmology based on the Friedmann solutions of general relativity, in which cosmic expansion is described as the dynamics of spacetime itself rather than as motion within a fixed background. In this framework, the initial singularity appears as a boundary of spacetime, and the concept of an exterior region is not part of the model.

Other cosmological constructions - such as the de Sitter universe or steady-state models - likewise provide mechanisms for redshift, but differ in their geometric interpretation and physical assumptions.

The geometric construction developed in the present work does not seek to revive purely kinematic cosmologies. Instead, it proposes a geometric correspondence between Schwarzschild and Friedmann–Robertson–Walker spacetimes within the framework of general relativity. In this sense, it aims to clarify how certain motion-based intuitions may be reinterpreted geometrically without departing from the relativistic description of spacetime.

The construction is characterized by three conceptual features:

1. Geometric origin without singularity
2. Recursive structure
3. Scale invariance

## 2 Introduction

### Proposition 1: Finite Representation of an Unbounded Radial Domain

There exist bijective mappings of the real numbers between 1 and  $\infty$  into the half-open interval of real numbers between 1 and 0 (open at 0), e.g.  $f(r) = \frac{1}{r}$  or of the real numbers between 0 and  $\infty$  into the half-open interval between 0 and 1 (open at 1), e.g.  $f(r) = \frac{r}{\sqrt{1+r^2}}$  (see Appendix 1).

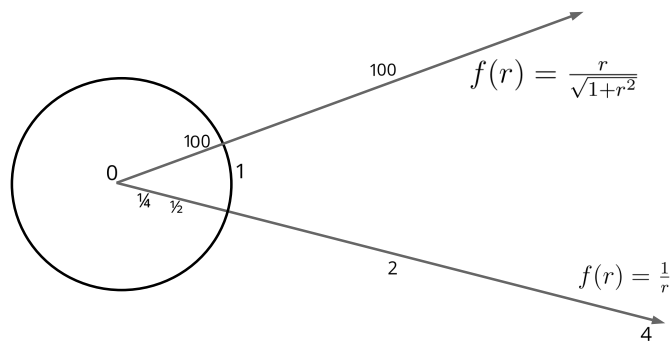


Figure 1:

This illustrates, at a purely geometric level, that infinite domains may be mapped into finite intervals by suitable coordinate transformations. Such mappings motivate the possibility of relating seemingly unbounded cosmological regions to bounded geometric configurations.

### Proposition 2: Relativity of Relative Scaling

When the relative size of two geometric objects changes, it is fundamentally indistinguishable which of the objects has changed its size. This symmetry under relative scaling is consistent with the geometric interpretation of gravitational collapse and the cosmological principle (compare with T. Fließbach, *Allgemeine Relativitätstheorie*, 6th edition, Equations 47.20 – Gravitationskollaps and 50.01 – Kosmologisches Prinzip).

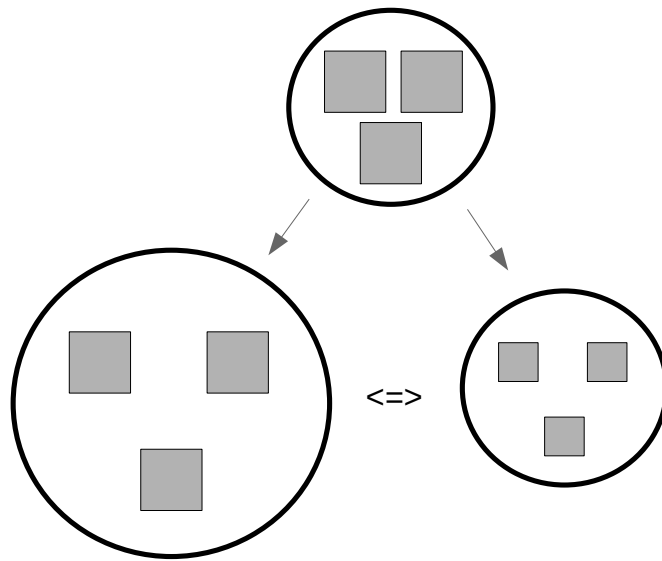


Figure 2:

This observation motivates a geometric perspective in which expansion and contraction may be regarded as relational descriptions rather than absolute processes. Within such a framework, it becomes meaningful to explore whether cosmological evolution can be represented as an interior geometric development compatible with an exterior Schwarzschild description.

### 3 Procedure

First, the hypersurface inside a light sphere is considered, consisting of the set of timelike geodesics that pass through the origin of the light (congruence emanating from a common origin event ).

Next, a distance function equivalent to that of Minkowski space is defined between the space-time events on these geodesics and the origin of the light.

The coordinate transformation from which the distance definition results is then applied to a hypothetical universe, whose initial state is assumed to have finite density and an extension greater than zero.

This involves specifying a compression that maps the unlimited interval of spacetime to a finite one.

It should be noted that, in the first step of this consideration, this hypothetical universe expands with  $c$  in Minkowski space.

The metric later derived from this distance definition is locally, but no longer globally, equivalent to Minkowski space, and can likewise be applied to the expansion of space.

In the next step, however, the hypersurface of a black hole at the moment of its formation - analogous to the hypersurface of the hypothetical universe - is considered, and the time span until the singularity (gravitational timescale) is defined as the time that the hypothetical universe would have needed to reach its initial state.

All spacetime events of the two hypersurfaces are then fully and bidirectionally mapped to each other, with continuous density at the transition.

Subsequently, the metric of the hypersurface is formally derived, transformed to the FRW-Metrik, and a (first special, later general) transformation from the exterior of the black hole to the universe inside it is given.

Finally, the resulting scale factor is determined and the local Hubble parameter is calculated.

Lastly, it is shown how the model could be parameterized across the recursive levels in a scale-invariant and observation-consistent manner.

Note: The discussion has shown that the procedure entails some fundamental difficulties, the resolution of which is not immediately apparent in four dimensions.

On the one hand, when globally mapping the curved spacetime of the exterior region of a black hole onto a universe located inside it — which is initially assumed to expand through

flat Minkowski space, but later, when Minkowski space is replaced by an expanding spacetime, could also be regarded as flat - one encounters the geometric impossibility of fully mapping of a region with non-zero Gaussian curvature onto a flat one. For example, it is impossible to map the world map of the Earth up to the North Pole (or South Pole) onto a Euclidean coordinate system.

On the other hand, it is not immediately clear what the difference is, in terms of measurable quantities, between a transformation through a singularity that is part of the model and one that is not included in the model.

Finally, one might ask to what extent the model offers advantages, given that an open interval at the event horizon also entails a singularity with respect to the coordinates. In this regard, it should be noted that the energy density on the matching hypersurface tends to zero, so that no surface layer with non-vanishing stress-energy is introduced, so this singularity can be regarded as purely mathematical. Although the coordinates become singular from the perspective of reference frames, no matter is actually present there, so the singularity carries no physical meaning.

To illustrate in an accessible way how the model deals with the mapping of curvature - and to explain the meaningfulness of the transformation - Appendix 3 first presents a two-dimensional example.

# 4 The 4-Dimensional Hypersphere

In the following, the term *hypersphere* shall always be understood to include its interior, not only the surface - same applies for the term *sphere*

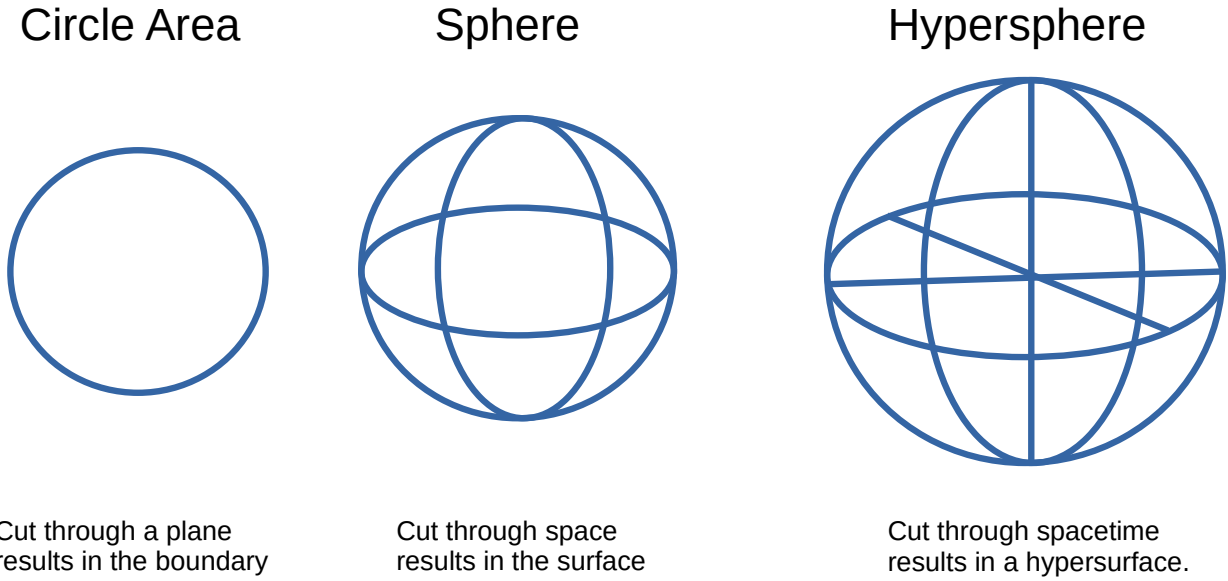


Figure 3:

In this construction, each point of an  $(n - 1)$ -dimensional base set is assigned two additional values in the  $n$ -th dimension, obtained from the sine and negative sine of the circle function applied to the distance from the center.

Properties:

- A)** All points on the boundary, the surface, or the hypersurface are at the same distance from the center.

- B)** There is only one center, which is not part of the boundary, the surface, or the hypersurface.
- C)** When moving along the boundary, the surface, or the hypersurface without a change of direction (angle preservation), one eventually returns to the starting point.

# 5 The Light Sphere in Space and Its Hypersurface in Spacetime

If two inertial observers,  $X$  and  $V$ , are at the same place at the same time where a sphere of light is emitted, the light expands in all directions from both observers at the speed  $c$ . This means that each observer perceives themselves as being at the center of the sphere of light in three-dimensional space.

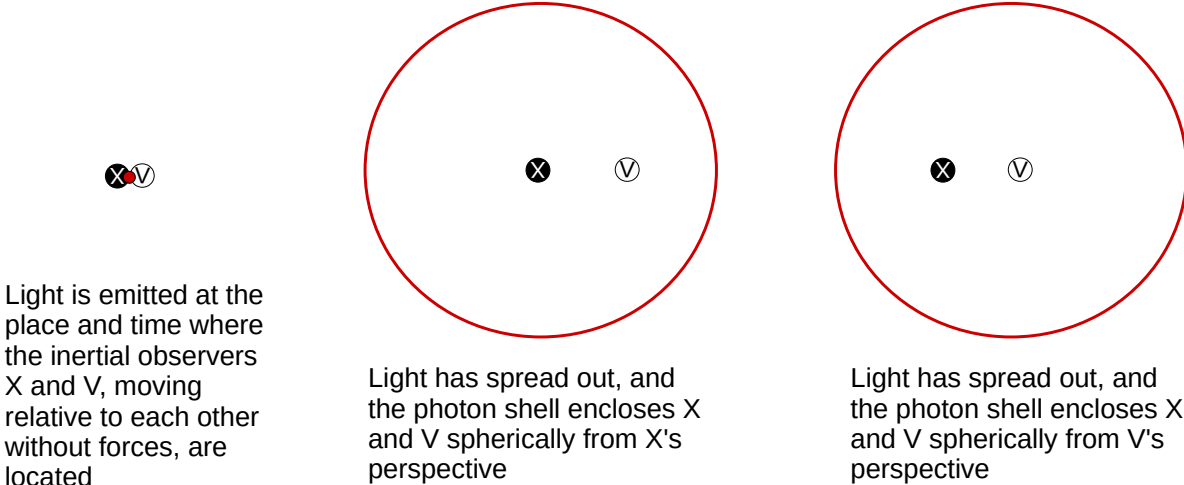


Figure 4:

However, the fact that a sphere can have only one center means that both observers are located (with a corresponding distance function to the center, in which, for example, no time passes for the photons) on the surface of a sphere of light in four-dimensional spacetime.

The photon shell thereby bounds a three-dimensional sphere in space and intersects the hypersurface of a four-dimensional hemispherical hypersphere in spacetime.

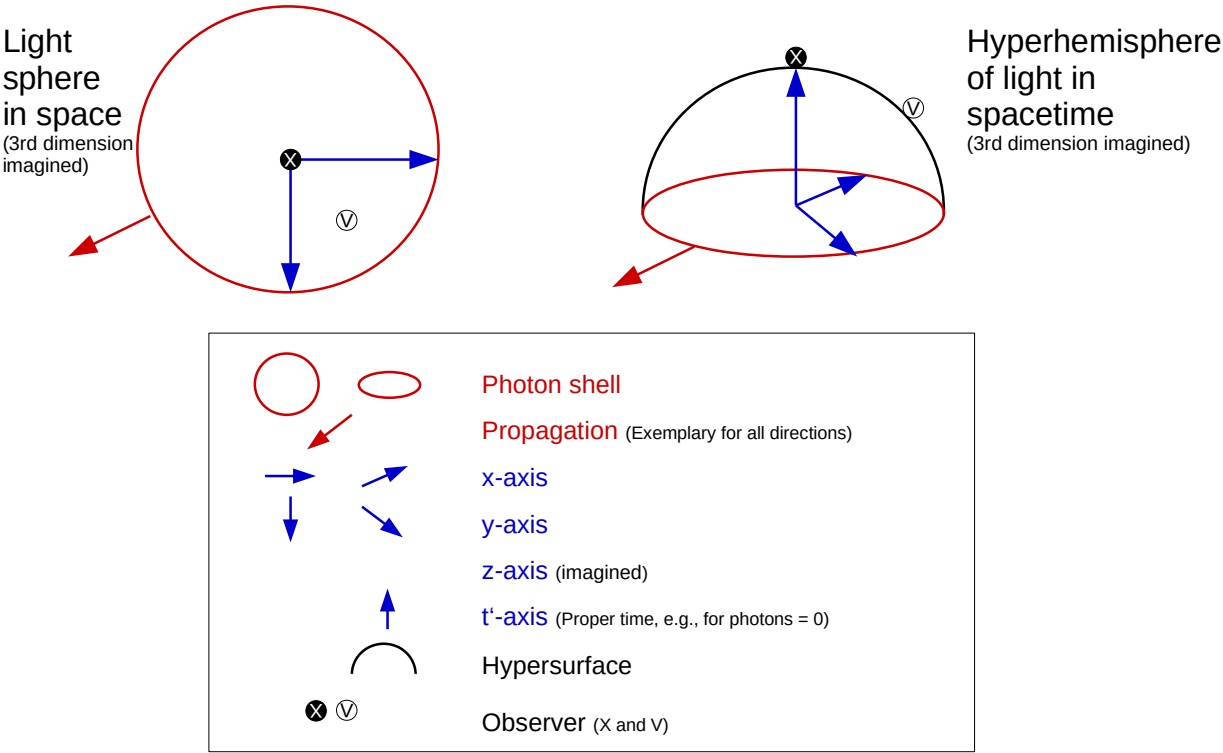


Figure 5:

According to the Lorentz transformation, properties A) and B) apply to the hypersphere of light in spacetime. Property C) would apply if light were also to expand into the past and one were to break through the light circle at superluminal speed in order to enter, within the same slice of spacetime (the same hypersurface), into a contracting sphere of light (a sphere that expands into the past is contracting from there into the future — see Reversal of Curvature Direction at the Event Horizon and Appendix 6).

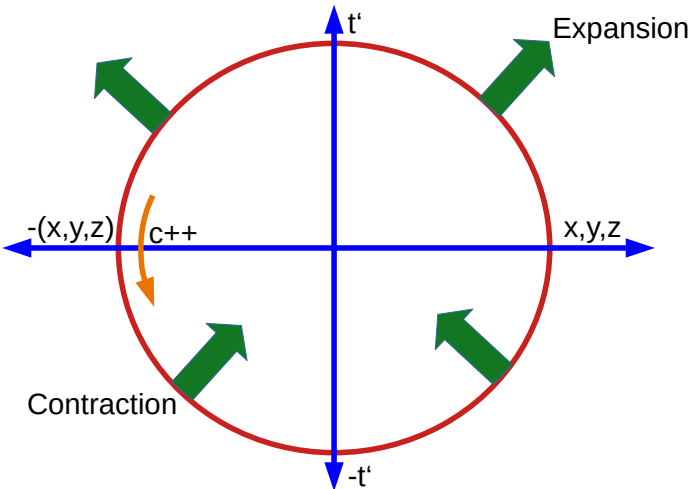


Figure 6:

### 6 Initialization

If one assumes the formula for time dilation of a moving object, as seen by a stationary observer, for the expansion of the universe:

$$t' = \sqrt{1 - \frac{v^2}{c^2}} t$$

This leads to the hypothesis that the universe is a hypersphere expanding at  $c$  (the speed of light), similar to a sphere of light.

For the representation, a distance function bijective to the distance function in Minkowski space is used:

$$D = \sqrt{r^2 + t'^2}$$

In this way, the usual description of spacetime events by the coordinates  $(x, y, z, t)$  is replaced by the equivalent coordinates  $(x, y, z, t')$ , where the coordinate axis  $t'$  represents the proper times of test particles expanding inertially from the origin.

The coordinate-system-independent description of the processes allows the choice of a suitable coordinate system for the representation.

The proper times of force-free particles ● that have moved from the origin to  $r$  at the time  $t' = 1$  in the observing system

$r$	$t'$
0	1
0,1	0,99
0,2	0,98
0,3	0,95
0,4	0,91
0,5	0,86
0,6	0,80
0,7	0,71
0,8	0,60
0,9	0,43
1	0

Lorentz Transformation

$$t' = \sqrt{1 - v^2/c^2}$$

for  $0 \leq r \leq 1 \Rightarrow v = rc$

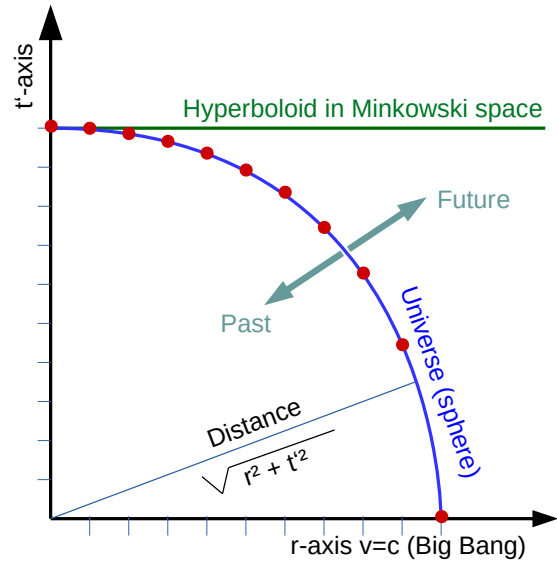
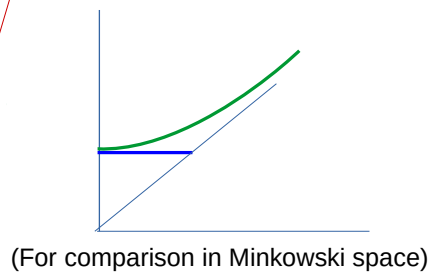


Figure 7:

The resulting model would exhibit the principal observed properties of the standard cosmological model: all galaxies recede from us, the farther away the faster (where the initially linear relation between distance and velocity is used for the sake of simplifying the model), and the origin (Big Bang) occurred everywhere.

To avoid the problem of a singularity in this model, one may assume that the universe did not originate from a singularity but from a state of maximal density ( $D_{\max}$ ). This would mean that, at its origin, the universe already had a finite extension (This implies the compression specified in Appendix 1).

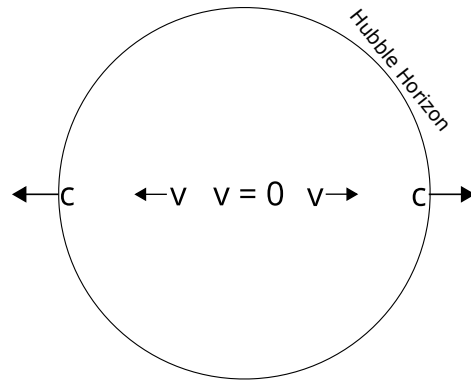


Figure 8: Initialization of the Universe at  $D_{\max}$

Analogously, a black hole at the moment of its formation can likewise be regarded as a hypersphere, which, however, contracts at  $c$ . In it, every distant point moves toward an arbitrary chosen center with a velocity  $v$  between 0 and  $c$  along the radius (the linear relation following from the model is precisely determined by the metric in the following; see Mathematical Derivation and Appendix 1).

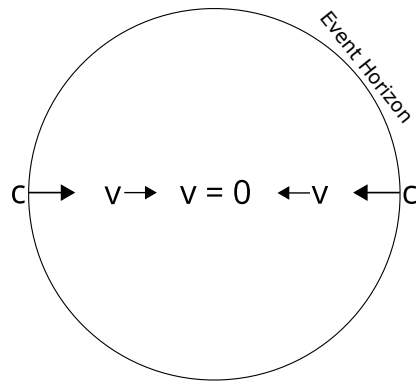


Figure 9: Formation of a Black Hole at  $D_{\max}$

One may now assume that this state of the black hole likewise represents its maximal density and that it cannot be further compressed.

At the moment the black hole forms, the distances inside begin to grow, so that from the inside it appears to expand at  $c$ , while from the outside it remains the same size (Proposition 2).

This means that the state of maximal density can be initialized via the Lorentz transformations of the different velocities between 0 and  $c$ . In this process, the velocities reverse their direction at the moment when  $D_{\max}$  arises.

This leads to the problem that velocities of equal magnitude cannot readily be mapped by the Lorentz transformations. On the one hand (Problem 1), for a hypothetical observer  $B2$  in the inertial system of observer  $B1$ , who (without loss of generality) regards himself as being at the center of the interval of maximal density, it would appear as if time were running backwards during the transition from the black hole to the universe (according to the Lorentz transformation: all clocks in the moving inertial system are shifted forward against the direction of motion and shifted backward with the direction of motion — relativity of simultaneity).

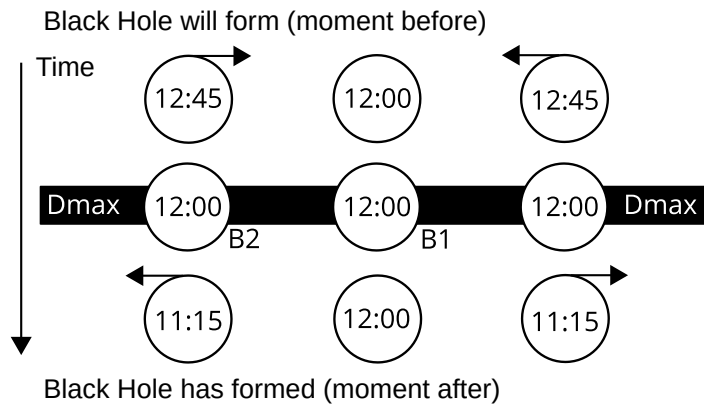


Figure 10:

On the other hand (Problem 2), the moving inertial systems at the position of  $B2$  would be in a state of density higher than  $D_{max}$ .

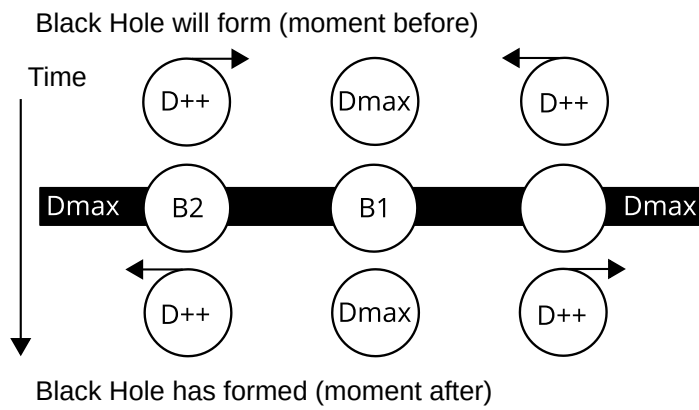


Figure 11:

## 6.1 Problem - 1

The model is idealized both within the Hubble radius and beyond the Schwarzschild radius, and it can be assumed for simplicity that the interior Schwarzschild solution can be extrapolated until the event horizon forms (Appendix 5).

Since the distant times are merely the coordinate times of the respective locally observed inertial systems (see Appendix 4), which, at the formation of the black hole, arise from the extrapolated interior Schwarzschild solution, the spacetime events are thus unambiguous up to this point. It is therefore sufficient, during the transition, to consider the respective physical state at this location, which, due to the symmetry of the Lorentz transformation, is identical (the clocks are shifted forward against the direction of motion by the same amount by which they are shifted backward with the direction of motion, and both densities are equally large — one already, the other not yet). This means that at this point there is no reversal of time, but merely a changing yet continuous perspective (one could say: first the glass is half full, then it is half empty, but in any case half of the content is in the glass).

The treatment of Problem 1 corresponds in Appendix 3 to the well-known identical temperature distribution on the sphere from the Tropic to the North Pole, or from the Tropic to the lower edge of the cylinder.

## 6.2 Problem - 2

Since observer  $B2$  is merely a hypothetical observer (in the inertial system of observer  $B1$ , no matter is located at the position of  $B2$ ), observer  $B1$  must move to the position of  $B2$  in order to determine the density there.

When considering the transition from the forming black hole to the black hole, one finds that the density at the position of  $B1$  is still less than  $D_{\max}$  ( $D^{--}$ ), while at the position of the hypothetical observer  $B2$  it is already greater than  $D_{\max}$  ( $D^{++}$ ). This means that, given continuous density, there must be a point in between with  $D_{\max}$ .

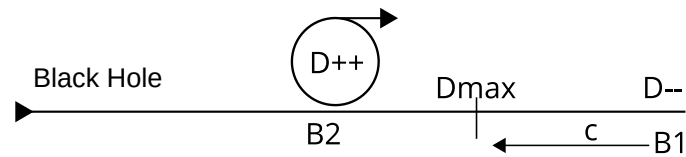


Figure 12:

$B1$  can move at most with the speed  $c$  and must pass  $D_{\max}$  in order to reach  $B2$ . From the moment it reaches  $D_{\max}$ , the directions of the velocities reverse, and the matter at the position of  $B2$  moves away from  $B1$ .

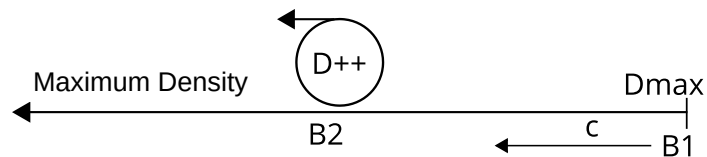


Figure 13:

According to the Lorentz transformation, it is not possible to reach the receding matter before its density is in a state smaller than  $D_{\max}$ .

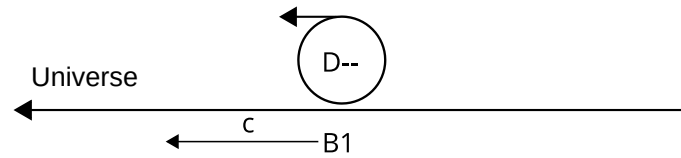


Figure 14:

The receding clock, which is set back due to the relativity of simultaneity, can never display an earlier time upon the arrival of  $B1$  than the one that, in its own inertial system, occurred at the position where  $B1$  started at  $D_{\max}$ . Although the clocks are mutually shifted according to the Lorentz transformation, time never runs backward for any observer.

The treatment of Problem 2 corresponds in Appendix 3 to the cutting and rejoining of the region with non-zero Gaussian curvature and the flat one at the Tropics.

## 7 Reversal of Curvature Direction at the Event Horizon

In examining the hypothesis that the universe is a hypersphere with respect to Definition C, the following can be established:

From the inertial frame of observer  $B1$ , there exists an acceleration  $a$  of constant magnitude with which the location receding at the speed  $c$ , that is, an event horizon (EH), is reached, and with which one subsequently returns to the same position in the original inertial frame. Although this acceleration exceeds  $c$  at the event horizon, the corresponding acceleration function can nevertheless be defined mathematically.

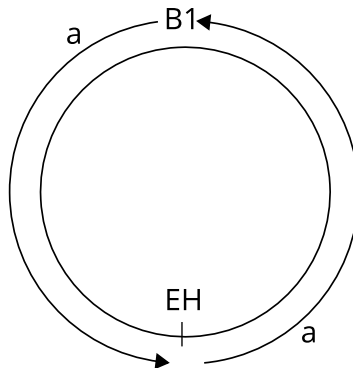


Figure 15:

However, when carrying out this function, one would find at two points on either side of the event horizon that in both cases an infinite physical acceleration in the opposite direction would be required in order to reach the event horizon.

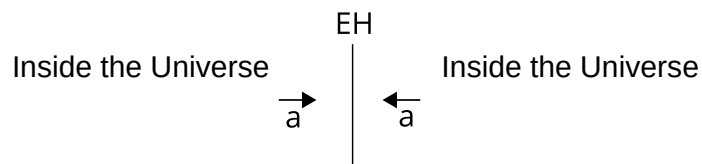


Figure 16:

This would contradict the notion that the universe is viewed from the other side of the event horizon as a black hole, because in that case one would physically have to accelerate with infinite acceleration in the same direction away from the event horizon, on the other side, in order not to reach it.

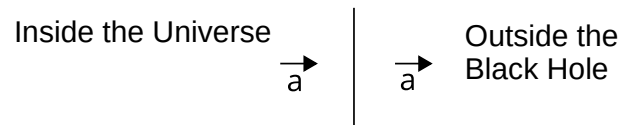


Figure 17:

In the representation used in this model, this corresponds to a reversal of the curvature direction at the event horizon, from which it follows that, in this model, the universe is a hemispherical hypersphere that is cut by the event horizon, with a hyperfunnel located on its opposite half.

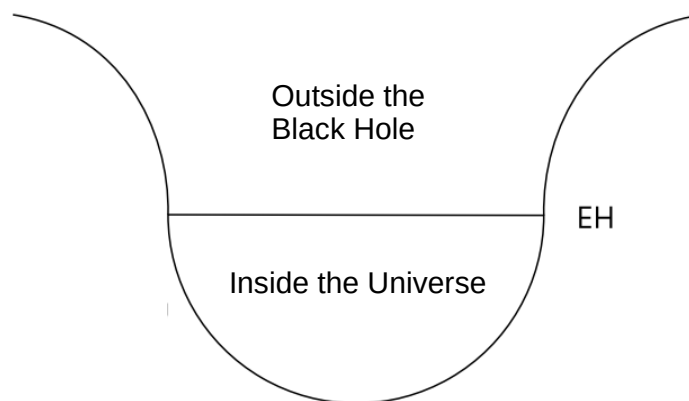


Figure 18:

In the model, the causal effect of the event horizon behaves in opposite ways from the inside and the outside. While it appears as a receding boundary in the expanding spacetime, it acts as a pull in the collapsing spacetime. This asymmetric behavior can be interpreted as an opposite curvature of the reference frames on this side compared to those on the far side, since the motion relative to the horizon changes its sign.

The term *hyperfunnel* refers to the four-dimensional spacetime structure that, analogous to the Flamm paraboloid, represents a funnel-shaped embedding of the Schwarzschild geometry. While the Flamm paraboloid provides an embedded representation of the spatial Schwarzschild geometry, the hyperfunnel is understood as the four-dimensional counterpart of this curvature.

This results in a change in the sign of the curvature at the inflection point between the funnel and the spherical surface.

## 8 Mathematical Derivation

The distance function used for the representation cannot be applied directly to determine the metric line element, because on the path from the origin to a hypothetical test particle receding inertially with velocity  $v$ , all other hypothetical test particles receding with velocities between 0 and  $v$  must be passed. For this reason, their respective proper lengths must also enter into the line element.

A universe expanding at  $c$  can, from the perspective of an arbitrary observer who regards themselves as being at the center, be represented schematically at the time  $T$  at which it originates with finite and maximal density as follows. In this representation, the infinite interval of all spacetime events of maximal density—that is, the corresponding hyperboloid in Minkowski space—is mapped onto the finite interval between 0 and  $r_s$ . At  $c$ , this interval is open (see Proposition 1, Problem 2 and Appendix 1).

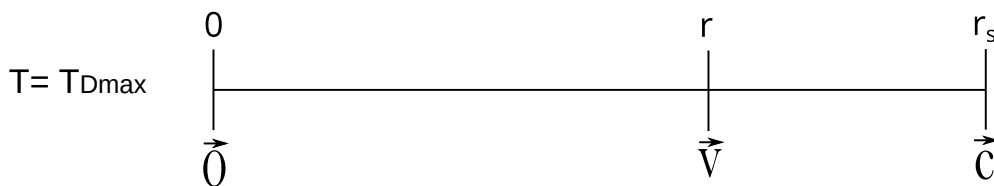


Figure 19:

Here,  $r_s$  denotes the distance to the event horizon. The expansion velocities  $v$  vary linearly between 0 and  $c$  (the linearity serving as a simplification of the model), and for all points  $r$  between 0 and  $r_s$  the following holds:

$$v = \frac{rc}{r_s}$$

Since proper time has been chosen as the global temporal structure—meaning that all space-time events are uniquely defined by the proper times of freely moving hypothetical test particles with velocities  $v(r)$  - the formula for the reading  $Z$  of a relativistically accelerated clock

$$Z = \int_0^T \sqrt{1 - \frac{v(t)^2}{c^2}} dt$$

and, respectively, the formula for the mileage  $K$  of a relativistically accelerating conveyor belt

$$K = \int_0^S \frac{1}{\sqrt{1 - \frac{v(s)^2}{c^2}}} ds$$

can be used as the basis for the path through the different expansion velocities of spacetime from a source at  $r=0$  to a spacetime event at  $T = T_{Dmax}$ . With  $v(r) = v$ , this yields the metric of the hypersurface of a hypersphere expanding at  $c$  in its initial state:

$$ds^2 = c^2 \left( 1 - \left( \frac{v}{c} \right)^2 \right) dt'^2 - \frac{1}{1 - \left( \frac{v}{c} \right)^2} dr^2$$

By substituting  $v = \frac{rc}{r_s}$ , one obtains at the time  $T = T_{Dmax}$ :

$$ds^2 = c^2 \left( 1 - \frac{r^2}{r_s^2} \right) dt'^2 - \frac{1}{1 - \frac{r^2}{r_s^2}} dr^2$$

This corresponds to a linear temporal transformation of the extrapolated interior Schwarzschild solution at the moment the event horizon forms (see Appendix 5).

Each point  $r$  on this side of  $r_s$  can now be bijectively assigned to a point  $R$  on the far side of  $r_s$ .

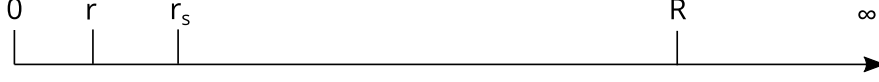


Figure 20:

According to Proposition 1, this yields—especially when scaled to  $r_s = 1$ , and in general with

$$r = \sqrt{\frac{r_s^3}{R}}$$

the exterior Schwarzschild solution as:

$$ds^2 = c^2 \left(1 - \frac{r_s}{R}\right) dt^2 - \frac{1}{1 - \frac{r_s}{R}} dR^2$$

For times  $t = t'$  at the position  $r = 0$ , which are greater than  $T = D_{\max}$ , the following holds when viewed from the inside, that is, on this side of  $r_s$ , due to the expansion

$$r = r(t) = r(0) + vt \quad \text{and} \quad r_s = r_s(t) = r_s(0) + ct$$

while  $r_s$  remains constant when viewed from the outside, that is, on the far side of  $r_s$  (Proposition 2).

This yields, for

$$R = \frac{r_s(t)^3}{r(t)^2}$$

again the metric of the hypersurface of a hypersphere expanding at  $c$ :

$$ds^2 = c^2 \left(1 - \frac{r(t)^2}{r_s(t)^2}\right) dt'^2 - \frac{1}{1 - \frac{r(t)^2}{r_s(t)^2}} dr^2$$

Note:

The change of differentials  $dr \rightarrow dR$  and  $dt' \rightarrow dt$  can be taken into account implicitly, since the relation between  $r$  and  $R$  is bijective and locally differentiable. Moreover, because all metrics considered are spherically symmetric, the angular coordinates of the polar coordinate system may, without loss of generality, be set to zero.

## 9 Transformation to the FRW Metric

As mentioned in Proposition 2, the FRW metric applies both to gravitational collapse and to the cosmological principle. Although it is the same metric, it describes different physical processes that give rise to entirely different spacetime events. In cosmological expansion, it is spacetime itself that expands, whereas in gravitational collapse the particles contract through a static spacetime in which the speed of light is not exceeded, even globally. This leads to the formation of an event horizon in the case of collapse, whereas in cosmological expansion it is possible to observe photons that were emitted from locations receding with superluminal expansion velocities.

In both cases, the description employs comoving coordinates, which can be transformed into static coordinates. When converting to the static coordinates of an observer, this automatically leads to a different interpretation of the time coordinate. In cosmological expansion, this is accompanied by cosmological redshift, which corresponds to cosmological time dilation. However, this effect is not relativistic in the sense of the Lorentz transformation, but arises geometrically from the change of the scale factor and formally corresponds to a classical Doppler effect. In contrast, when the FRW metric is used to describe motion through a static space, relativistic time dilation must be employed for the transformation to static coordinates.

The FRW metric has the disadvantage that it is not defined up to the boundary. Moreover, it contains a singularity. In contrast, the derived metric of a hemispherical hypersphere expanding at  $c$  has the advantage that the boundary can be taken into account in the form of a purely coordinate singularity. Furthermore, it has the advantage that it does not contain a singularity.

For the transformation, it is therefore sufficient to consider

$$T_{Dmax} = t > 0, 0 \leq r(t) < r_s(t)$$

The transformation is presented in seven steps:

- 1) The spacetime events on the timelike geodesics passing through the origin, which serve as the basis of the derivation, can be described in Minkowski space - by applying the FRW metric to motion through a static space - analogously to gravitational collapse, but with an opposite  $a(t)$  (expansion instead of contraction) and without gravity, i.e. purely according to special relativity.
- 2) A coordinate transformation of these events is performed, in which, from the perspective of an arbitrary but fixed static observer, the proper time of the freely comoving test particle is used instead of the Minkowski-space time coordinate to label the event. In this way, exactly the same events are described using a different time coordinate.
- 3) Each spacetime event that lies on a timelike geodesic passing through the origin like-

wise lies on a hyperboloid in Minkowski space. One of these hyperboloids, corresponding to  $T = D_{\max}$ , is transformed according to Proposition 1 (see Appendix 1) into the coordinate system underlying the derived metric.

4) The metric obtained in this way at the initialization time is now evolved, yielding the metric of the hypersurface of a hypersphere expanding at  $c$ :

$$ds^2 = c^2 \left( 1 - \frac{r(t)^2}{r_s(t)^2} \right) dt'^2 - \frac{1}{1 - \frac{r(t)^2}{r_s(t)^2}} dr^2$$

5) When applying the expansion to the initial metric, the time-dependent change of the line element would formally have to be taken into account via the temporal variation of  $r_s$  or  $r$ . This is cumbersome, since a concrete calculation would require explicitly considering the differently contracted lengths, and it is also unnecessary because, due to the linearity  $v = cr$ , the change in  $r_s$  affects the line element in the same way regardless of its location. That is, if  $r_s$  doubles, both the strongly contracted lengths at the boundary and the weakly contracted lengths near the center double as well; likewise, the dilated times at the boundary change in the same proportion relative to the total time as the weakly dilated times near the center. This, in turn, means that the temporal change can be represented in the form of a scale factor  $a$ , which acts directly on the line element of the initial metric:

$$ds^2 = c^2 a(t)^2 \left( 1 - \frac{r^2}{r_s^2} \right) dt'^2 - a(t)^2 \frac{1}{\left( 1 - \frac{r^2}{r_s^2} \right)} dr^2$$

6) The transformation described in step 2) is now reversed and the term corresponding to time dilation disappears:

$$ds^2 = c^2 a(t)^2 dt'^2 - a(t)^2 \frac{1}{\left( 1 - \frac{r^2}{r_s^2} \right)} dr^2$$

7) than the resulting conformal time, which arises from the introduction of the scale factor:

$$dt' = dt/a(t)$$

is transformed into the corresponding cosmological time, to receive the FRW-Metric with a curvature parameter  $k = 1$ :

$$ds^2 = c^2 dt^2 - a(t)^2 \frac{1}{\left( 1 - \frac{r^2}{r_s^2} \right)} dr^2$$

## 10 The Scale Factor

In the model, for large distances, even without spacetime expansion - that is, when freely moving objects recede from one another through space - the relativistic addition of velocities leads to an ever smaller increase in redshift while producing an ever larger luminosity distance

$D_L(z)$ . This behavior would correspond to a cosmological constant ( $\Lambda > 0$ ) in the standard model. In this context,  $r_s$  can serve as a parameter to compute the observed relation from the model.

In order to obtain scale invariance (such that the recursive structures always appear the same), additional parameters, for example the expansion of space, may possibly be taken into account.

Since at large distances the separations are contracted according to the metric, the scale factor  $a(t', r)$  depends not only on time but also on the expansion velocity at the location under consideration. This means that the ruler is initialized differently at each location, but then expands uniformly with time along the  $t'$ -axis (since the consideration always proceeds from an arbitrary location regarded as being at the center, for which

$$v = 0 \Rightarrow t = t',$$

one may, for simplicity, write  $a(t)$  instead of  $a(t')$ , and correspondingly  $a(t, r)$  instead of  $a(t', r)$ ).



Since

$$v(r_0) = \frac{r_0 c}{r_s}$$

it follows that

$$r = r_0 + \frac{r_0}{r_s} ct$$

Thus,

$$r_0 a(t) = r_0 + \frac{r_0}{r_s} ct$$

and therefore

$$a(t) = 1 + \frac{ct}{r_s}$$

From this, the Hubble parameter  $H(t)$  follows as

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{\frac{c}{r_s}}{1 + \frac{c}{r_s} t}$$

## 12 Scale Invariance and Observation-Conform Parameterization

In the analysis of the scale factor as well as in the determination of the Hubble parameter, it becomes apparent that  $r_s$  parametrizes both  $D_L(z)$  and  $H$ . Since  $r_s$  is directly associated with mass, it is natural that  $r_s$  is also responsible for structure formation (i.e., how rapidly galaxies or black holes form out of the plasma that produces the 3 K cosmic background radiation).

To derive scale invariance, two black holes may be considered:

BH-1 has the mass  $M = 1$ ,  
the Schwarzschild radius  $r_s = 1$ ,  
and the characteristic proper time  $T_{D_{\max}} = 1$  (the time BH-1 would require in the exterior spacetime to reach the singularity).

BH-2 has the mass  $M = 2$ ,  
the Schwarzschild radius  $r_s = 2$ ,  
and the characteristic proper time  $T_{D_{\max}} = 2$ .

If absolute homogeneity is assumed at  $D_{\max}$  — that is, all spacetime points are equivalent in every respect and no two spacetime points can be distinguished — then the larger mass

can be compensated by a correspondingly higher speed of light  $c$  in the interior:

$$c \propto r_s$$

In the example, light would propagate twice as fast inside BH–2 as inside BH–1. As a result, the two hemispherical hyperspheres are indistinguishable from the inside, and scale invariance is achieved.

A numerical calculation suggests that inserting approximately the present-day Hubble radius as  $r_s$  into the model can reproduce the observed values of  $D_L(z)$ , whereas a smaller value of  $r_s$  reproduces the observed local Hubble parameter. If the simplifying assumption  $v(r) \propto r$  were abandoned (an intuitive choice would be  $v(r) = \frac{r(1+\alpha r)}{1+\alpha r^2}$  where  $\alpha$  parameterizes the nonlinearity, but for  $r_s = 1$ , the structure of the model allows monotonically increasing functions with  $v(0) = 0$  and  $v(r_s) = c$ ), additional degrees of freedom would become available to determine a suitable  $r_s$ , since this would decouple the local Hubble parameter from the large-distance behavior of  $D_L(z)$ .

When applying the metric to an expanding spacetime, a significantly smaller  $r_s$  can reproduce the observations, since the additional expansion can affect  $D_L(z)$  and the Hubble parameter in opposite ways. In this model, the speed of light between mutually receding objects is not constant with respect to coordinate distance, because space itself expands between them. Nevertheless, the identical speed  $c$  of all objects relative to the event horizon - and thus the representation of the universe as a hemispherical hypersphere - remains possible. This is because the expansion of space, unlike light, does not need to propagate through the expanding space, but is a property of spacetime itself. The metric used here is derived from the assumption that the universe constitutes a four-dimensional hemispherical hypersphere. Owing to the symmetry of the Lorentz transformation with respect to the event horizon, it can be applied consistently in this model, even though the global relative velocity between comoving objects does not correspond to the local speed of light.

A rough estimate of structure formation using the Press–Schechter model likewise indicates a possible agreement with observations in this case. Structure formation proceeds somewhat more rapidly, which might potentially explain the existence of unusually large cosmic voids.

## 13 Distinction from the Inflationary Phase

Homogeneity / Background

Since widely separated regions originated from the same initial state, they exhibit a similar structure today without having had to interact with one another.

Anisotropy

The anisotropy can be explained by slight temperature fluctuations, with regard to quantum

effects arising shortly after the initial conditions.

#### Flatness

Since the curvature is caused by the compression of the representation in proper time, it cannot be measured locally from within.

#### Magnetic Monopoles

Within the model, magnetism originates solely from the relativistic description of electric currents.

## 14 Mass / Radius Ratio as a Dynamic Degree of Freedom

The following section departs from the established framework of the recursive model and presents speculative considerations that arise from possible extensions.

A further possibility for variation within the model is provided by the fact that the moment at which the black hole forms from the outside and the moment at which the universe forms from the inside need not be identical, since after its formation the black hole retains its fundamental structure as a hemispherical hypersphere during contraction. This allows both spatial and temporal freedom for a transformation of the (virtual) region of spacetime with density greater than  $D_{\max}$  (see Problem 1 and Problem 2), as well as for relocating the scaling function associated with the growth of black holes into this region. Furthermore, the origin of the factor  $\frac{1}{2}$  mentioned in Appendix 5 could likewise be shifted to this domain.

## 15 Appendix 1 - Transformation of the hyperboloids from Minkowski space

If a black hole forms with a Schwarzschild radius  $r_s$ , then, when regarded as a hypersphere contracting at the speed  $c$ , it would require the time  $r_s/c$  to contract to a radius of zero.

Analogously, the universe, considered as a hypersphere expanding at  $c$ , would require the same time to expand to the radius  $r_s$ .

However, since a black hole does not continue to contract beyond the state of maximal density, but instead gives rise to the universe at maximal density, this time can be used as  $T_{D_{\max}}$  for initialization, and the following relation holds:

$$r_s = cT_{D_{\max}} \Leftrightarrow T_{D_{\max}} = \frac{r_s}{c}$$

Since this time  $T_{D_{\max}}$  furthermore corresponds to the proper time associated with the velocity  $v$  of receding objects, the elapsed time  $T$  at the origin for  $s = 0$  satisfies:

$$T_{Dmax} = T\sqrt{1 - \frac{v^2}{c^2}} \Leftrightarrow T = \frac{T_{Dmax}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For the distance  $s$  of the objects from the origin, this yields:

$$s = vT = v \frac{T_{Dmax}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v}{c} \frac{r_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Since these velocities - when mapped onto the finite interval between 0 and  $r_s$  - vary proportionally to the speed of light between 0 and  $r_s$ , it further follows that:

$$v = \frac{rc}{r_s}$$

Thus,

$$s = \frac{r}{r_s} \frac{r_s}{\sqrt{1 - \frac{(rc/r_s)^2}{c^2}}} = \frac{r}{\sqrt{1 - \left(\frac{r}{r_s}\right)^2}}$$

Equivalently,

$$r = \frac{s}{\sqrt{1 + \left(\frac{s}{r_s}\right)^2}}$$

## 16 Appendix 2 - General transformation of the Schwarzschild metric

In Figure 4, it becomes clear that the center of the photon sphere in space depends on the observer's inertial frame. This applies to all observers, both inside and outside the photon sphere.

If one now wants to shift point  $r$  to the center in Figure 20, i.e., to adopt the perspective of an observer previously located at point  $r$ , who then considers themselves to be at the center, the expansion velocity  $v$  at location  $r$ , which exactly corresponds to the escape velocity  $v$  at location  $R$ , must be compensated.

Therefore, the general mapping is:  $r \rightarrow R$  (with  $v$ ).

Or equivalently:  $R$  (with  $v$ )  $\rightarrow r$ .

This means that when assigning the metrics at location  $R$ , one should no longer consider the external Schwarzschild metric, but rather that of the free-falling observer who moves with the escape velocity at  $R$  relative to the mass center.

In this sense, the transformation from  $R$  (with  $v$ ) to  $r$ , in accordance with the information loss at the event horizon, is unidirectional. That is, from the outside, a center can be defined

according to the state of motion, but from the inside, no center can be determined, since  $v$  cannot be determined.

## 17 Appendix 3 - A Two-Dimensional Analogue

The surface of a sphere, similar to the Earth's surface, is to be mapped onto the surface of a cylinder (here, the mapping of the upper hemisphere onto the lower half of the cylinder is considered). The measured quantity in terms of coordinates is the temperature  $t(l)$ , which is assumed to be highest at the equator and to decrease continuously and monotonically along the length  $l$  toward the poles (for simplicity, linearly). It turns out that the temperature above a certain latitude (tropic) is so low that no one can ever reach it to take measurements, let alone return from there. This gives rise to the idea that the world actually ends at that point. However, since it is inconceivable for a world to simply stop at any point, it must be connected at that location to another world—the surface of the cylinder. There, the temperature is also highest in the center and decreases continuously and monotonically toward the edges (e.g., linearly), and, just like on the sphere, there is also a region beyond which it becomes too cold to perform measurements or return from (tropics).

Where one world ends, the other begins, and vice versa. This means that the regions where it is colder than above the corresponding latitude on the sphere (or below the corresponding latitude on the cylinder) are not part of either world and do not exist.



cating spacetime events with a density greater than  $D_{\max}$ ) and the solution of Problem 1 (transforming the measured values, i.e., the density, over a truncated and thus non-existent singularity). The truncated region of spacetime can be considered as imaginary spacetime, since the speed of light is formally exceeded in this region (square root of minus one).

## **18 Appendix 4 - Apparent discontinuity as a coordinate artifact**

The reason why a change in coordinate time does not affect physical processes becomes clear when one considers the following: Suppose the Earth were suddenly to rotate in the opposite direction around its own axis. In that case, for example, in Berlin, one hour before noon, the clock would have to be adjusted (in a discontinuous manner) from 11:00 a.m. to 1:00 p.m., without this affecting the passage of time for the physical processes themselves.

## 19 Appendix 5 - Transition to the inner Schwarzschild metric

The interior Schwarzschild solution strictly applies only to static configurations in hydrostatic equilibrium. During gravitational collapse this condition is no longer fulfilled, and the space-time becomes genuinely time-dependent. Nevertheless, at any given instant before horizon formation, the configuration may be formally compared to a static interior solution with the same instantaneous mass and radius. Such a comparison does not represent a true solution of the field equations but serves as a useful reference state, analogous to the use of the instantaneous rest frame in accelerated motion.

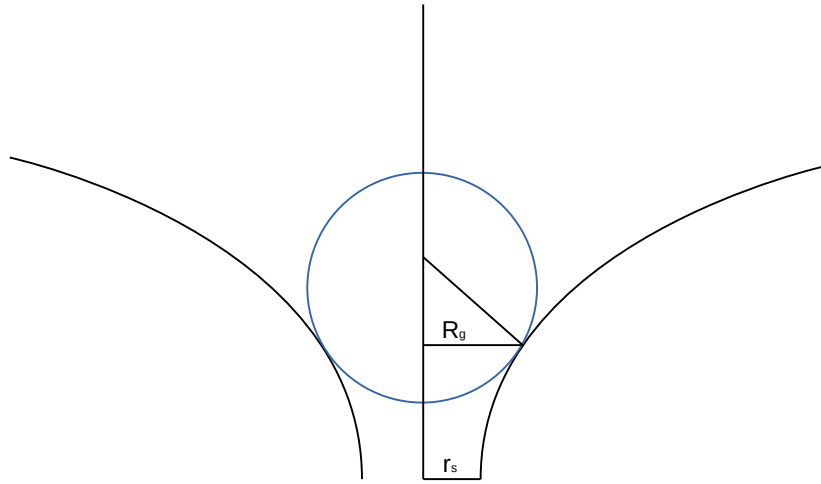


Figure 23:

The interior Schwarzschild solution:

$$ds^2 = c^2 \left( \frac{3}{2} \sqrt{1 - \frac{r_s}{R_g}} - \frac{1}{2} \sqrt{1 - \frac{r_s r^2}{R_g^3}} \right)^2 dt^2 - \frac{1}{1 - \frac{r_s r^2}{R_g^3}} dr^2$$

with  $R_g = r_s$  yields the derived metric at the time  $T = D_{\max}$ :

$$ds^2 = c^2 \left( 1 - \frac{r^2}{r_s^2} \right) dt'^2 - \frac{1}{1 - \frac{r^2}{r_s^2}} dr^2$$

However, it is slower by a temporal factor.

This means:

If, in this hypothetical model, a fictitious person were located inside the circle of Proposition 1 or Proposition-2, this person would appear to shrink from the outside, and time there would progress faster overall. The apparent location of this fictitious person, as seen from the outside, would depend on the acceleration of the outer observer's state of motion, and there would always be external observers for whom the fictitious person appears to be at the center.

If one wants to know how this appears to the fictitious person, it can be calculated according to Appendix 1.

The absence of contradictory spacetime events is ensured by the metrics, which allow all processes in the interior and exterior to be described unambiguously and which smoothly join at the boundary.

## 20 Appendix 6 - The complete hypersphere

A light sphere in spacetime (4D) is not a fully spherical hyper-sphere, but a hemisphere, because it is directed only into the future – light propagates only causally, i.e., for  $t \geq 0$  positively along the time axis.

In this model, however - due to the special structure of the event horizon (EH) - the same EH can be understood as a section of a complete hyper-sphere, because it is causally touched both from the inside and the outside, albeit with reversed curvature / time orientation (see Chapter 5):

From the outside: Spacetime contracts toward the EH  $\rightarrow$  as in a collapse,  
From the inside: Spacetime expands away from the EH  $\rightarrow$  as in a Big Bang.

It follows that:

The EH forms the intersecting surface of a symmetric hyper-sphere, which "emerges" from the inside with reversed time direction and is embedded from the outside in a complete hyperfunnel (see Chapter 7).

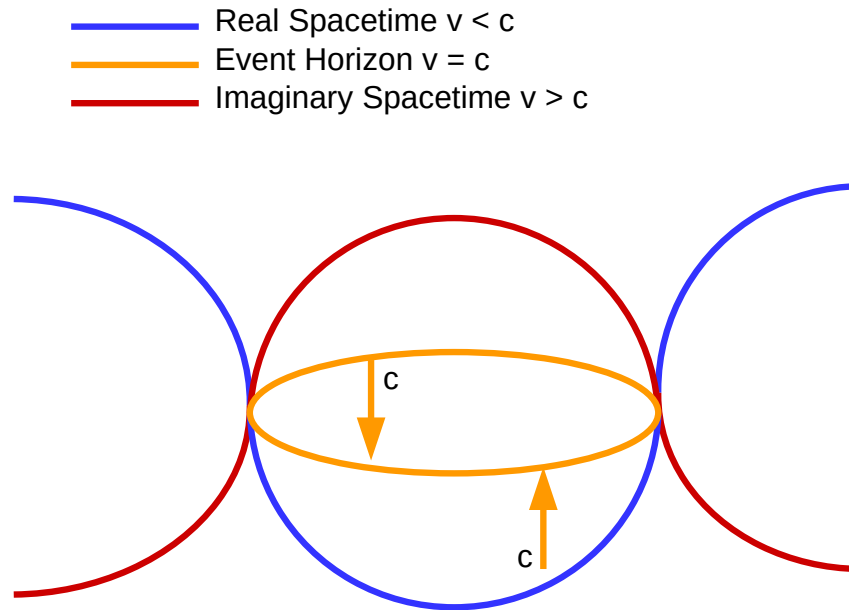


Figure 24:

## 21 Sources / References / Additional Literature

Standard terminology and notation for relativistic cosmology follow the conventional references [2–3].

Additional Literature [4–6] is provided for further reading.

1) Source - T. Fließbach, Allgemeine Relativitätstheorie, 6th ed., Spektrum Akademischer Verlag, Heidelberg (2008).

2) Reference - K. Schwarzschild, Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (1916).

3) Reference - A. Friedmann, Über die Krümmung des Raumes, Zeitschrift für Physik 10, 377–386 (1922).

4) Additional Literature - E. Hubble, A relation between distance and radial velocity among extra-galactic nebulae, Proc. Natl. Acad. Sci. 15, 168–173 (1929).

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